BUILDING MATHEMATICS MODELING FOR SOLVED TRANSPORTATION PROBLEMS AND OPTIMIZING WITH MORE FOR FEWER ALGORITHMS IN THE BUSINESS COMMUNITY

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Abstract

Mathematical modeling supports the development of the business world and the industrial world, especially in transportation (Widana, 2020). Many emerging algorithms are combined with the adoption of changes in the form of real problems modeled into mathematical problems. The modeling aims to optimize to produce the most optimal objective function value based on the formed constraints. Optimization of the mathematical model can be assisted by several optimization algorithms, such as the More for Less Algorithm. The algorithm, as a form of looping, is based on the constraint function with the specified parameters until it reaches the final objective function, namely optimization related to time and cost(Muftikhali et al., 2018). In general, it is hoped that supply and demand will be balanced in transportation. However, in reality, often in field conditions, mixed transportation problems result in non-optimal costs. The study results show that the More for Less Algorithm can provide the most optimal solution according to the allocation of factory requests to shops with the minimum cost. The dual variable matrix index is positive, so the solution to the transportation problem with mixed constraints is called optimal. The solutions obtained in the transportation case study with mixed constraint functions in the field of the Amarta Bakery industry community business are as follows:$x_{11} = 90$, $x_{12} = 90$, $x_{22} = 90$, $x_{23} = 90$, dan $x_{33} = 90$ with a total cost of 580 (in units of money). Combinations - factory-to-store combinations for companies are interpreted as first factory at the first store with an allocation of 90 product units, first factory at the second shop with an allocation of 110 product units, second

factory at the second shop with an allocation of 0 product units, factory the second factory at the third store with an allocation of 70 units of product, and the third factory at the third store with an allocation of 0 units of product. The number zero means that to carry out efficiency and optimization according to mixed constraints, it is necessary to make conditions where factories do not distribute to shops.

Keywords: Mathematics Modeling, More for Less, Transportation, Mix Problem, Optimizing

I. INTRODUCTION

A transportation problem is a special form of linear programming problem that is interconnected and has correlation and regression(Gamal et al., 2003). Most transportation cases experience problems because they have to assign one item from various sources to a predetermined allocation. Many cases of transportation problems can be found in various industrial sectors, communication networks, planning, delivery services, etc. When changes in delivery allocation placement change, it will also affect the difference in cost and time(Gamal et al., 2003). Seeing that the delivery route can be determined optimally, the transportation problem becomes very interesting to solve as an effort to minimize costs.

The problem of transportation is minimizing costs and balancing supply and demand. Looking at the various real events that are happening, the balance between supply and demand very often overlaps according to trends, seasons, and other factors, which causes supply to be greater than demand or vice versa. The problem of market imbalance becomes an obstacle that will also affect transportation imbalances.

The transportation simplex method can solve cases of transportation problems in general or specifically(Nurmayanti & Sudrajat, 2021). Determination of the initial solution can be determined using the northwestern corner method, minimum cost method, and Vogel. As a form of optimization and search algorithms to solve transportation problems, the authors are interested in using the more-for-less algorithm with mixed constraints(Munirah & Subanar, 2017). So, this writing, will be determined mathematical modeling to form real problems of transportation problems with constraints/mixed constraint functions that are formed into mathematical problems so that the results of the model can be determined which solutions will be optimized with the more for less algorithm as the optimization in finding the smallest fitness(Simanullang &Budhayanti, 2003).

II. RESEARCH AND METHOD

Transportation problems with mixed constraint functions can be determined by modeling real problems as mathematical problems about the objective function using the Vogel method. Mathematical modeling begins with determining the initial solution using the index of 2 (two) matrix variables and the more for less algorithm. Pandian and Natarajan (2010) state that to obtain the smallest boundary problem, you can do a crossover/crossover of inequality on the mixed constraint function and convert it to an equation. The right-hand side of the inequality is the minimum value of supply and demand. From this concept, a mathematical model can be written as a solution to the smallest boundary problem as follows(Kristiana Sinaga et al., 2013):

$$min\ Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$

With the constraint function (Equation 1):

$$\sum_{j=1}^{n} x_{ij} = s_i, i \in U$$

$$\sum_{j=1}^{n} x_{ij} = 0, i \in V$$

$$\sum_{j=1}^{n} x_{ij} = s_i, i \in W$$

$$\sum_{i=1}^{m} x_{ij} = d_j, j \in Q$$

$$\sum_{i=1}^{m} x_{ij} = 0, j \in T$$

$$\sum_{i=1}^{n} x_{ij} = d_j, j \in S$$

$$X_{ij} \geq 0, i = 1,2,\dots,m \text{ , dan } j = 1,2,\dots,n$$

With:

u : The set is separate from the set $U \cup V \cup W = 1,2,\dots,m$

V : The set is separate from the set $U \cup V \cup W = 1,2,\dots,m$

W : The set is separate from the set $U \cup V \cup W = 1,2, \ldots, m$

Q A separate set from the set Q $\cup T \cup S = 1,2, \ldots, n$

Q A separate set from the set Q $\cup T \cup S = 1,2, \ldots, n$

S A separate set from the set Q $\cup T \cup S = 1,2, \ldots, n$

Bronson, R. (1996) states that in determining the initial solution, you can use the northwestern corner method, the minimum cost method, and Vogel. In several studies and practices, the Vogel method can provide a solution that is closest to the optimal value to provide the optimal solution(M & Subanar, 2017). The following are the steps in determining the initial solution with the Vogel method:

1. Calculate fines/plate in each row and column by finding the difference between the highest and lowest values.
2. Analyze the row and column with the highest penalty. Move the allocation as much as possible to the place with the lowest cost in accordance with supply and demand. Mark the rows and columns that have been fulfilled. If there is more than one row and column directly, one can be selected by adding a mark randomly to the selected row and column. The remaining unselected rows and columns can be assigned a value of 0 (zero) and do not need to be included in subsequent calculations(M & Subanar, 2017).
3. Condition selection process
   a. Stop the process when there is one unmarked row or column left
   b. If one row or column remains that does not have *marks*, give the variable basis to the row or column with the minimum cost
   c. If some rows and columns do not have a mark but have a value of 0 (zero), give the variable basis a value of 0 (zero) with the minimum cost method. Then stop.
   d. If conditions a, b, and c do not occur, return to the first step, calculating the penalty for rows and columns that do not have *marks*, and return to Step number 2.

After the initial solution is formed using Vogel, the dual variable index matrix is formed dan , which corresponds with the matrix cells and the sum. The sum result has several conditions; if the result is negative, a new transportation problem can be formed with a mixed constraint function(i, j) UI dan v_j(Pandian & Natarajan, 2010).

The results of the solution calculation will vary greatly depending on the initial solution formed or *the crossovers* specified(Massalesse, 2019). In this study, researchers used an algorithm *more for less* as a form of optimization in finding the objective function or *fitness* of the minimum. In summary, the process of determining optimization with the more-for-less algorithm with mixed constraint functions is as follows(Acharya et al., 2015):

1. Model real transportation problems with mixed constraint function into mathematical problem models with the help of linear programming.
2. The form of a transportation problem matrix table with mixed constraint functions in a transportation table
3. Form the least limit problem of the transportation problem with mixed constraints with Steps:
   a. Swap all the inequalities in the constraint function where on each right-hand side is the smallest value of*supply*And*demand.*The general form of the equation is contained in Equation 1 above.
   b. Create a transportation problem table with the constraint function according to the 1st equation.
4. Determine the initial solution.
   a. Analysis *supply* And*demand*has have been balanced; if there is an imbalance, provide a dummy variable to run the method.
   b. Calculate the solution by Vogel's method.
5. Solution optimization test

   Sum the variables determined from the initial solution matrix index. If the sum value is negative, a transportation problem must be formed with a new mixed constraint function according to the algorithm *'s rules, which means more for less.*
6. Process*Looping*

   Re-run the calculations from Steps 1 to 4 to see the solution's optimality. If there are no more negative values, the solution is the local optimal solution.$u_i + v_j$

III. FINDINGS AND DISCUSSION

The problem is taken in the food industry "Amarta Bakery". The industry is located in Tulungagung Regency and is spread across various areas within the regency, which are quite far apart. In this case, Amarta Bakery has 3 production sites and 3 storage areas connected to stores in six different locations. This study focused on cakes with a shelf life of more than 1-2 weeks. Factory 1 is capable of providing 60 units of cakes. Factory 2 was able to produce 70 units due to the increased availability of materials. While at factory 3, *maintenance* on one of the tools caused a reduction in production capacity with a maximum of only 100 units. If the demand for shop 1 is 90 units, the demand for shop 2 must be at least 110 units, and the shop for at least 70 units due to problems with existing equipment. In this case, information obtained that the cost of shipping one type of goods per unit from the factory to the shop is presented in Table 1.

Table 1. Shipping Costs for Each Unit

Track	Shop 1	Shop 2	Shop 3
Factory 1	2	3	4
Factory 2	6	3	1
Factory 3	8	9	2

Determining the optimal solution to the industrial problem can be solved by an algorithm *more or less* according to the steps previously described.

Step 1. Model real transportation problems with mixed constraint function into mathematical problem models with the help of linear programming as follows:

$$\min z = 2x_{11} + 3x_{12} + 4x_{13} + 6x_{21} + 3x_{22} + 1x_{23+} + 8x_{31} + 9x_{32+}2x_{33}$$

Constraint functions that occur are demand constraints, supply constraints, non-negative constraints, and integer constraints. The following details the value of each obstacle experienced

Demand Constraints:

$$x_{11} + x_{12} + x_{13} = 90$$

$$x_{21} + x_{22} + x_{23} \geq 110$$

$$x_{31} + x_{32} + x_{33} \leq 70$$

Offer Constraints:

$$x_{11} + x_{12} + x_{13} = 60$$
$$x_{21} + x_{22} + x_{23} \geq 70$$
$$x_{31} + x_{32} + x_{33} \leq 100$$

Non-negated constraints and integer constraints:

$$x_{ij} \geq 0 \text{ dan integer.}$$

Step 2. The form of a transportation problem matrix table with mixed constraint functions in a transportation table.

Table 2. Transportation of Amarta Bakery with Mixed Constraint Functions

	Shop 1	Shop 2	Shop 3	*Supply*
Factory 1	2	3	4	$= 60$
Factory 2	6	3	1	≥ 70
Factory 3	8	9	2	≤ 100
Requests	$= 90$	≥ 110	≤ 70	

Step 3. The form of the least limit problem of the transportation problem with mixed constraints.

a. Mathematical models

$$\min z = 2x_{11} + 3x_{12} + 4x_{13} + 6x_{21} + 3x_{22} + 1x_{23+} + 8x_{31} + 9x_{32+}2x_{33}$$

Because the third factory experienced problems, the execution of the constraints on the third equation was as follows:

Request constraints:

$$x_{11} + x_{12} + x_{13} = 90$$
$$x_{21} + x_{22} + x_{23} = 110$$
$$x_{31} + x_{32} + x_{33} = 0$$

Bid Constraints

$$x_{11} + x_{12} + x_{13} = 60$$

$$x_{21} + x_{22} + x_{23} = 70$$

$$x_{31} + x_{32} + x_{33} = 0$$

Non-negated constraints and integer constraints:

$$x_{ij} \geq 0 \text{ dan integer.}$$

b. Update the transportation problem table with the smallest boundary problem

Table 3. The Transportation Problem with the Least Boundary Problem

	Shop 1	Shop 2	Shop 3	*Supply*
Factory 1	2	3	4	= 60
Factory 2	6	3	1	= 70
Factory 3	8	9	2	= 0
Requests	= 90	= 110	= 0	200 / 130

Step 4. Initial solution determination

Equilibrium analysis of transportation problems, if there is an imbalance, it is necessary to add a dummy variable. In the above case, the number of requests is greater than supply. To balance this problem, a dummy variable is added with a total inventory of 70 at a cost of 0 (zero). The transport equilibrium table is as follows:

Table 4. Equilibrium Transportation Problem with Dummy Variable

	Shop 1	Shop 2	Shop 3	*Supply*
Factory 1	2	3	4	60
Factory 2	6	3	1	70
Factory 3	8	9	2	0
Dummy Factory	0	0	0	70
Requests	90	110	0	200

After the form of the transportation problem is balanced, the penalty calculation is carried out to determine the initial solution using the Vogel method. Results as follows:

Table 5. Equilibrium Transportation Problem with Dummy Variable

	Shop 1	Shop 2	Shop 3	*Supply*	Line Penalty
Factory 1	2	3	4	60	1
Factory 2	6	3	1	70	2
Factory 3	8	9	2	0	6
Dummy Factory	0	0	0	70	0
Requests	90	110	0		
Column Penalty	2	3	1		

The penalty calculation is based on the smallest value in the first column, namely 2 for, for the second column, namely 3 for, so that the row penalty is the difference between the minimum of the second column, namely. All rows and columns are calculated in the same way. So that the biggest penalty is worth 6 in row 3. Because the third row has the largest penalty, an allocation with a minimum cost equal to the table adjustment at the third factory has been made; column 3 is deleted. The adjustment and calculation of the value of the new table are shown in the following table:$C_{ij} C_{11} C_{12} 3-2 = 1 C_{33} x_{33} = \min\{0,0\}$.

Table 6. Vogel's Method and Allocation

	Shop 1	Shop 2	Shop 3	*Supply*	Line Penalty
Factory 1	2	3	4	60	1
Factory 2	6	3	1	70	2
Factory 3	8	9	0 2	0	-
Dummy Factory	0	70 0	0	0	0
Requests	90	40	0		
Column Penalty	2	0	1		

The equilibrium table has been formed, then choose column 2 as the second because it has a penalty of 3. Allocation to the matrix because it has the (smallest) value. The amount that can be allocated is. This allocation will delete the 4th row and reduce the demand in columns 2 to 40. This process is repeated until supply and demand are met, and the initial solution is obtained from the Vogel method, namely: and other variables are 0 (zero) or non-basic, as in the following table.$x_{42} c_{ij} = 0$ $\min z = \{70{,}110\}$ $x_{11} = 60; x_{21} = 30; x_{22} = 40; x_{23} = 0; x_{33} = 0; x_{42} = 70;$

Table 7. Vogel's Initial Solution

	Shop 1	Shop 2	Shop 3	*Supply*
Factory 1	60 [2]	[3]	[4]	60
Factory 2	30 [6]	40 [3]	0 [1]	70
Factory 3	[8]	[9]	0 [2]	0
Dummy Factory	[0]	70 [0]	[0]	70
Requests	90	110	0	

After the initial solution with Vogel is obtained, the value of the objective function/fitness can be calculated as follows:

$$\min z = 2 \times (60) + 6 \times (30) + 3 \times (110) + 1 \times (0) + 2 \times (0) = 630$$

The initial solution inequalities obtained on Vogel are presented in the following table:

Table 8. Initial Table of Transportation Problems with Mixed Constraint Functions

	Shop 1	Shop 2	Shop 3	*Supply*
Factory 1	60 [2]	[3]	[4]	$= 60$
Factory 2	30 [6]	110 [3]	0 [1]	≥ 70
Factory 3	[8]	[9]	0 [2]	≤ 100
Requests	$= 90$	≥ 110	≤ 70	

Step 5. Solution Optimization Test

In this step, we will test the optimization of the solution obtained from the simplex table. As seen in Table 8 as an initial solution, the basis variable can be determined from the sum. So the following results are obtained (equation -3):$z_{ij} = u_i + v_j u_i + v_j = c_{ij}$

$$u_1 + v_1 = 2$$
$$u_2 + v_1 = 6$$
$$u_2 + v_2 = 3$$
$$u_2 + v_3 = 1$$
$$u_3 + v_3 = 2$$

Make an exchange on one of the variables randomly. Suppose selected, and then the following results are obtained:$u_2 = 0$

$$u_1 = -4, u_2 = 0, u_3 = 1, v_1 = 6, v_2 = 3, v_3 = 1$$

Next, a dual variable matrix is formed that corresponds to the cell matrix that has been obtained and then added together.$u_i\ dan\ v_j (i, j) u_i$ dan v_j

Table 9. Dual Variable Matrix Index

Dual Variables	v_1	v_2	v_3	UI
u_1	2	-1	-3	-4
u_2	6	3	1	0
u_3	7	4	2	1
v_j	6	3	1	

Step 6. Looping Process

Because there are negative values in the first row, second, and third column indexes, a new transportation problem must be solved with mixed constraints according to the more-for-less algorithm with the following mathematical model:

$$\min z = 2x_{11} + 3x_{12} + 4x_{13} + 6x_{21} + 3x_{22} + 1x_{23+} + 8x_{31} + 9x_{32+} 2x_{33}$$

Constraint functions are demand, supply, non-negative, and integer constraints. The following details the value of each obstacle experienced

Demand Constraints:

$$x_{11} + x_{12} + x_{13} = 90$$
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$$x_{31} + x_{32} + x_{33} = 70$$

Offer Constraints:

$$x_{11} + x_{12} + x_{13} \geq 60$$
$$x_{21} + x_{22} + x_{23} \geq 70$$
$$x_{31} + x_{32} + x_{33} \leq 100$$

Non-negated constraints and integer constraints:

$$x_{ij} \geq 0 \text{ dan integer.}$$

So, we get a table of transportation problems with mixed constraints in the table below:

Table 10. The Transport Problem with the New Mixed Constraint Function

	Shop 1	Shop 2	Shop 3	*Supply*
Factory 1	2	3	4	$\geq$ 60
Factory 2	6	3	1	$\geq$ 70
Factory 3	8	9	2	$\leq$ 100
Requests	$\geq$ 90	$\geq$ 110	$=$ 70	

The looping process is carried out in Steps 2 to 4 so that the optimal solution is obtained according to the optimality test dan in the cells shown in the following table:(i, j)

Table 11 Positive Dual Variable Matrix

Dual Variables	v_1	v_2	v_3	UI
u_1	2	3	1	0
u_2	2	3	1	0
u_3	3	4	2	1
v_j	2	3	1	

In Table 11, all dual variable matrix indices have positive values. Then the solution is called the optimal solution for the new mixed constraints so that the solution to the transportation constraints is obtained$x_{11} = 90, x_{12} = 110, x_{22} = 0, x_{23} = 70, x_{33} = 0$ with a total cost of 580.

IV. CONCLUSIONS AND SUGGESTIONS

Conclusion

According to the analysis and calculation results of the optimized mathematical model with the more-for-less algorithm, the industrial problems at Amarta Bakery can be handled properly. The results of the analysis show a positive solution which is called the optimal solution, namely the allocation of factories and shops can be interpreted as factory 1 to Warehouse 1 of 90, factory 1 to store 2 of 110, factory 2 to store 3 of 70 with transportation costs of 580 (in currency units).

Suggestions

In future research, various mixed problems can be added as a function of mixed constraints. Moreover, the application of optimization algorithms can be hybridized with several other algorithms, such as harmony algorithms, genetic algorithms, and other optimization algorithms, to get a more optimal solution in terms of analysis, time, and cost.

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